



Indirect reciprocity with private, noisy, and incomplete information

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Indirect reciprocity is a mechanism for cooperation based on shared moral systems and individual reputations. It assumes that members of a community routinely observe and assess each other and that they use this information to decide who is good or bad, and who deserves cooperation. When information is transmitted publicly, such that all community members agree on each other's reputation, previous research has highlighted eight crucial moral systems. These "leading-eight" strategies can maintain cooperation and resist invasion by defectors. However, in real populations individuals often hold their own private views of others. Once two individuals disagree about their opinion of some third party, they may also see its subsequent actions in a different light. Their opinions may further diverge over time. Herein, we explore indirect reciprocity when information transmission is private and noisy. We find that in the presence of perception errors, most leading-eight strategies cease to be stable. Even if a leading-eight strategy evolves, cooperation rates may drop considerably when errors are common. Our research highlights the role of reliable information and synchronized reputations to maintain stable moral systems.

cooperation | indirect reciprocity | social norms | evolutionary game theory

Humans treat their reputations as a form of social capital (1–3). They strategically invest into their good reputation when their benevolent actions are widely observed (4–6), which in turn makes them more likely to receive benefits in subsequent interactions (7–12). Reputations undergo constant changes in time. They are affected by rumors and gossip (13), which themselves can spread in a population and develop a life of their own. Evolutionary game theory explores how good reputations are acquired and how they affect subsequent behaviors, using the framework of indirect reciprocity (14–17). This framework assumes that members of a population routinely observe and assess each other's social interactions. Whether a given action is perceived as good depends on the action itself, the context, and the social norm used by the population. Behaviors that yield a good reputation in one society may be condemned in others. A crucial question thus becomes: Which social norms are most conducive to maintain cooperation in a population?

Different social norms can be ordered according to their complexity (18) and according to the information that is required to assess a given action (19, 20). According to "first-order norms," the interpretation of an action depends only on the action itself. When a donor interacts with a recipient in a social dilemma, the donor's reputation improves if she cooperates, whereas her reputation drops if she defects (21–26). According to "second-order norms," the interpretation of an action additionally depends on the reputation of the recipient. The recipient's reputation provides the context of the interaction. It allows observers to distinguish between justified and unjustified defections (27–29). For example, the standing strategy considers it wrongful only to defect against well-reputed recipients; donors who defect against bad recipients do not suffer from

an impaired reputation (30). According to "third-order norms," observers need to additionally take the donor's reputation into account. In this way, assessment rules of higher order are increasingly able to give a more nuanced interpretation of a donor's action, but they also require observers to store and process more information.

When subjects are restricted to binary norms, such that reputations are either "good" or "bad," an exhaustive search shows there are eight third-order norms that maintain cooperation (20, 31). These "leading-eight strategies" are summarized in Table 1, and we refer to them as L1–L8. None of them is exclusively based on first-order information, whereas two of them (called "simple standing" and "stern judging," refs. 32 and 33) require second-order information only. There are several universal characteristics that all leading-eight strategies share. For example, against a recipient with a good reputation, a donor who cooperates should always obtain a good reputation, whereas a donor who defects should gain a bad reputation. The norms differ, however, in how they assess actions toward bad recipients. Whereas some norms allow good donors to preserve their good standing when they cooperate with a bad recipient, other norms disincentivize such behaviors.

Ohtsuki and Iwasa (20, 31) have shown that if all members of a population adopt a leading-eight strategy, stable cooperation can emerge. Their model, however, assumes that the players' images are synchronized; two population members would always agree on the current reputation of some third population member. The assumption of publicly available and synchronized information

Significance

Indirect reciprocity explores how humans act when their reputation is at stake, and which social norms they use to assess the actions of others. A crucial question in indirect reciprocity is which social norms can maintain stable cooperation in a society. Past research has highlighted eight such norms, called "leading-eight" strategies. This past research, however, is based on the assumption that all relevant information about other population members is publicly available and that everyone agrees on who is good or bad. Instead, here we explore the reputation dynamics when information is private and noisy. We show that under these conditions, most leading-eight strategies fail to evolve. Those leading-eight strategies that do evolve are unable to sustain full cooperation.

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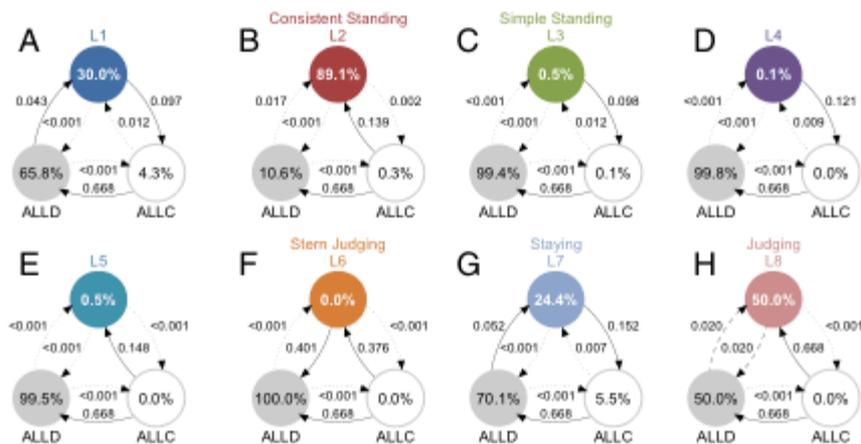


Fig. 3. Most of the leading-eight strategies are disfavored in the presence of perception errors. We simulated the evolutionary dynamics when each of the leading-eight strategies competes with ALLC and ALLD. These simulations assume that, over time, players tend to imitate coplayers with more profitable strategies and that they occasionally explore random strategies (*Materials and Methods*). The numbers within the circles represent the abundance of the respective strategy in the selection–mutation equilibrium. The numbers close to the arrows represent the fixation probability of a single mutant into the given resident strategy. We use solid lines for the arrows to depict a fixation probability that exceeds the neutral probability $1/N$, and we use dotted lines if the fixation probability is smaller than $1/N$. In four cases, we find that ALLD is predominant (C–F). In one case (H), the leading-eight strategy coexists with ALLD but without any cooperation. In the remaining cases (A, B, and G), we find that L1 and L7 are played with moderate frequencies, but only populations that have access to L2 (“consistent standing”) settle at the leading-eight strategy. Parameters: Population size $N = 50$, benefit $b = 5$, cost $c = 1$, strength of selection $s = 1$, error rate $\epsilon = 0.05$, observation probability $q = 0.9$, in the limit of rare mutations $\mu \rightarrow 0$.

There are only three scenarios in Fig. 3 that allow for positive cooperation rates. The corresponding leading-eight strategies are L1, L2 (“consistent standing”), and L7 (“staying,” ref. 45). For L1 and L7, the evolutionary dynamics take the form of a rock–scissors–paper cycle (46–50). The leading-eight strategy can be invaded by ALLC, which gives rise to ALLD, which in turn leads back to the leading-eight strategy. Because ALLD is most robust in this cycle, the leading-eight strategies are played in less than one-third of the time (Fig. 3A and G).

Only consistent standing, L2, is able to compete with ALLC and ALLD in a direct comparison (Fig. 3B). Under consistent standing, there is a unique action in each possible situation that allows a donor to obtain a good standing. For example, when a good donor meets a bad recipient, the donor keeps her good standing by defecting, but loses it by cooperating. Compared with stern judging, which has a similar property (18), consistent standing incentivizes cooperation more strongly. When two bad players interact, the correct decision according to consistent standing is to cooperate, whereas a stern player would defect (Table 1).

Nevertheless, we find that even when consistent standing is common, the average cooperation rate in the population rarely exceeds 65%. To show this, we repeated the previous evolutionary simulations for the eight scenarios while varying the benefit-to-cost ratio, the error rate, and the observation probability (Fig. 4). These simulations confirm that five of the leading-eight strategies cannot maintain any cooperation when competing with ALLC and ALLD. Only for L1, L2, and L7 are average cooperation rates positive, reaching a maximum for intermediate benefit-to-cost ratios (Fig. 4A). If the benefit-to-cost ratio is too low, we find that each of these leading-eight strategies can be invaded by ALLD, whereas if the ratio is too high, ALLC can invade (*SI Appendix, Fig. S5*). In between, consistent standing may outperform ALLC and ALLD, but in the presence of noise it does not yield high cooperation rates against itself. Even if all interactions are observed ($q = 1$), cooperation rates in a homogeneous L2 population drop below 70% once the error rate exceeds 5% (*SI Appendix, Fig. S4*). Our analytical results in *SI Appendix* suggest that while L2 populations always recover from single disagreements, it may take them a substantial time to do so, during

which further errors may accumulate. As a result, whereas L2 seems most robust when coevolving with ALLC and ALLD, it is unable to maintain full cooperation. Furthermore, additional simulation results suggest that even if L2 is able to resist invasion by ALLC and ALLD, it may be invaded by mutant strategies that differ in only one bit from L2 (*SI Appendix, Fig. S6*).

So far, we have assumed that mutations are rare, such that populations are typically homogeneous. Experimental evidence, however, suggests that there is considerable variation in the social norms used by subjects (4, 7–11). While some subjects are best classified as unconditional defectors, others act as unconditional cooperators or use more sophisticated higher-order strategies (11). In agreement with these experimental studies, there is theoretical evidence that some leading-eight strategies like L7 may form stable coexistences with ALLC (36). In *SI Appendix, Figs. S7–S9*, we present further evolutionary results for higher mutation rates, in which such coexistences are possible.

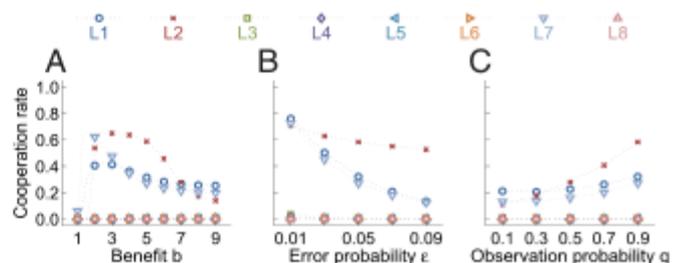


Fig. 4. Noise can prevent the evolution of full cooperation even if leading-eight strategies evolve. We repeated the evolutionary simulations in Fig. 3, but varying (A) the benefit of cooperation, (B) the error rate, and (C) the observation probability. The graph shows the average cooperation rate for each scenario in the selection–mutation equilibrium. This cooperation rate depends on how abundant each strategy is in equilibrium and on how much cooperation each strategy yields against itself in the presence of noise. For five of the eight scenarios, cooperation rates remain low across the considered parameter range. Only the three other leading-eight strategies can persist in the population, but even then cooperation rates typically remain below 70%. We use the same baseline parameters as in Fig. 3.

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