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Date: Mon, 04 Dec 2017 12:59:01 +0000

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why is transfinite recursion a good model for understanding – the proof that the result is well-defined uses transfinite induction. Let F denote a (class) function F to be defined on the ordinals. The idea now is that, in defining $F(\alpha)$ for an unspecified ordinal α, one may assume that $F(\beta)$ is already defined for all $\beta < \alpha$ and thus give a formula for $F(\alpha)$ in terms of these $F(\beta)$. It then follows by transfinite induction that there is one and only one function satisfying the recursion formula up to and including α.

(more will be given later): define function F by letting $F(\alpha)$ be the smallest ordinal not in the set $\{F(\beta) \mid \beta < \alpha\}$, that is, the set consisting of all $F(\beta)$ for $\beta < \alpha$. This definition assumes the $F(\beta)$ known in the very process of defining F; this apparent vicious circle is exactly what definition by transfinite recursion permits. In fact, $F(0)$ makes sense since there is no ordinal $\beta < 0$, and the set $\{F(\beta) \mid \beta < 0\}$ is empty. So $F(0)$ is equal to 0 (the smallest ordinal of all). Now that $F(0)$ is known, the definition applied to $F(1)$ makes sense (it is the smallest ordinal not in the singleton set $\{F(0)\} = \{0\}$), and so on .

it sort of says an approximation to truth. by reduction. alternately we can add other dimensions.