Project Description

The University of Miami is a private university located in Coral Gables, Florida, with about 10,000 undergraduate and 5,000 graduate and professional students. It is the premier research and teaching university in south Florida with several well-established programs in its Department of Physics, even though the Department has less than twenty full-time physics faculty. In view of its unique position to so large a part of Florida, we believe it is very important to support programs in physics at the University of Miami. Further information about the University and the Department can be found at *umiami.edu*.

This is a proposal to conduct research in the theory of elementary particles and fields at the University of Miami's Department of Physics where we are the two senior particle theorists, having joined the Department in 1988 (TC) and 1986 (LM). While the Physics Department has consistently supported our research efforts during the last three decades, additional outside support is critically needed for us to carry out research.

Since the 1970s we have each made significant contributions to the study of renormalization theory, supersymmetric field theories, supergravity, string and membrane theory, symmetries and quantum effects for nonlinear sigma models, integrable models, quantum groups, deformation quantization, generalized Hamiltonian mechanics, and most recently, galileon models and strings in 3D. At present we have a small but focussed group actively working on several fundamental problems in those areas. More specifically, the problems involve the relation of evolution in discrete and continuous spacetimes, branched Hamiltonian systems, galileon models and singularities in general relativity in various spacetime dimensions, and supersymmetric models of semions, anyons, and strings in three-dimensional spacetime. We consider these to be important projects with possible novel applications to real-world systems. We use both new and well-established mathematical methods to achieve a deeper theoretical understanding of the physics. We believe our methods will provide the means to discover and understand many key developments in the future. We have various well-recognized collaborators at other institutions, as is evident in our list of publications. We usually have graduate students working with us, and we take great care to mentor them.

We regularly teach and advise large numbers of undergraduates and a few graduate students in the College of Arts and Sciences at the University, and we perform service work for both the Department and the College. When we do not teach, usually during the summer months, we frequently visit research laboratories (such as CERN near Geneva, or JINR in Dubna) and other research institutions (such as the IAS in Princeton, NJ, or the Isaac Newton Institute in Cambridge, UK). These visits contribute to our teaching as well as our research insofar as they enable us to maintain a current knowledge and perspective of contemporary physics — essentials for teaching and mentoring.

We also regularly organize meetings as a service to the physics community. In 1990, 1991, 1996, and 2003 we co-organized and participated in successful conferences on quantum groups and their uses in physics, on integrable models, on duality, and on the generalized dynamics of branes. The proceedings of the first two meetings were published as books by World Scientific while the proceedings for the last two meetings are only available on the Argonne internet site.

Ten years ago, under the sponsorship of the University's Department of Physics and College of Arts and Sciences, we initiated a new series of annual winter meetings on elementary particles, astrophysics, and cosmology to supplant the previous series of "Coral Gables conferences" (1964–2003). This brings to south Florida a large number of research physicists, both theoretical and experimental, and gives all graduate students in our Department the opportunity to interact with them. The first ten meetings were very successful and well-received by the physics community, with approximately 100 participants coming to each meeting, many travelling internationally. We have organized an eleventh conference for about 80 participants, to take place December 2014. We expect this new series of meetings to continue for the foreseeable future.

Comments on the proposed budget

We have requested a minimum of support to carry out our research without the need to shoulder greater teaching and administrative burdens for personal financial reasons.

We have also requested a modest amount of support for a long-time collaborator, Cosmas Zachos, who has been, in our opinion, prematurely severed from research support at Argonne National Laboratory. We believe his forced early retirement from his ANL position was not warranted, especially given his current productivity and active participation in the field of elementary particle theory. Our own research efforts stand to be impaired if Dr Zachos does not have some support to continue collaborative work. Thus we propose to hire him as a senior research Scientist at the University of Miami, provided there is sufficient funding received for the research proposed here.

Results from Prior NSF Support

During the last twelve years, since the summer of 2003, we have received a total of $448,000 from NSF (awards PHY- 0303550, 0555603, 0802988, 0855386, 0937580, and 1214521). Supported by these grants, we have completed and published a significant body of results, including that listed below, and we have a substantial amount of work in progress. For the relation of all this previous research to the future research proposed here, please see the section to follow, on Proposed Research.

Our most recent NSF award was $120,000 for *Elementary Particle Theory*, PHY-1214521, beginning in the summer of 2012 and active until June 30, 2015 (although all funds have been expended by now). All publications and other evidence of research resulting from that latest NSF award are given by the lists to follow, for dates after June 2012.

Recent activities (including travel, conferences, and seminars)

We regularly visit other institutions each year and we attend theoretical physics conferences. We maintain highly visible national and international profiles. Some activities from the last five years are as follows.

Professor Curtright's recent activities: (slides for all my talks are available online)

• Seminar, "Galileons and Gravity", University of Groningen, 20 May 2014.
• Participant, Solvay Institute Workshop "Fast is Beautiful: Supersymmetry and Strings in a Null Frame" 14-15 May 2014, ULB, Brussels.
• Seminar, "Galileons and Gravity", College of William and Mary, 15 April 2014.
• Participant, "50 Years of Quarks and Color", University of Maryland – College Park, 11-12 April 2014.
• Visiting Scientist, Argonne National Lab., 13-18 March 2014.
• Co-organizer and participant, *Miami 2013*, 12-18 Dec 2013
• Participant, "Quark 50", Caltech, 9-10 Dec 2013.
• Seminar, "Branched Hamiltonians and Supersymmetry", Wigner Research Centre for Physics, 12 Nov 2013.
• Participant, "Wigner 111", Hungarian Academy of Sciences, Budapest, 11-13 Nov 2013.
• Seminar, "Galileons and Gravity", Washington University, St Louis, 17 Oct 2013.
• Seminar, "Galileons and Gravity", University of Iowa, 26 July 2013.

• Visiting Scientist, Argonne National Lab., July 2013.
• Seminar, "Five Talks with Myron", Bander Symposium, University of California - Irvine, 8 June 2013.
• Seminar, "Galileons Unchained", University of Missouri - Columbia, 12 March 2013.
• Co-organizer and participant, *Miami 2012*, 13-20 Dec 2012.
• Seminar, "Galileons and Naked Singularities", FAU, 14 November 2012.
• Participant and session chair, Solvay Institute Workshop "The Quantum Quest: A Fascinating Journey" 5-7 November 2012, ULB, Brussels.
• Participant, "Review of Discovery Physics Results from ICHEP", Argonne National Lab., 17 July 2012.
• Visiting Scientist, Argonne National Lab., July 2012.
• Colloquium, "*P̄hase Space*", Kansas University, Lawrence, 30 April 2012.
• Seminar, "Conjugation, Cycles and the C-theorem", University of North Carolina, Chapel Hill, 20 March 2012.
• Colloquium, "*P̄hase Space*", University of North Carolina, Chapel Hill, 19 March 2012.
• Co-organizer and participant, *Miami 2011*, 15-20 Dec 2011.
• Seminar, "Functional Composition, Conjugation, Cycles, and the C-theorem", Vanderbilt University, 28 October 2011.
• Visiting Scientist, Center for Theoretical Physics, MIT, 16 August 2011.
• Seminar, "Functional Methods", University of Iowa, 2 June 2011.
• Co-organizer and participant, *Miami 2010*, 14-19 Dec 2010.
• Seminar, "Functional Conjugation Methods: Renormalization Group", Feza Gürsey Institute, Istanbul, 5 November 2010.
• Seminar, "Functional Conjugation Methods: Dynamical Systems", Physics Dept., Boğaziçi University, Istanbul, 2 November 2010.
• Visiting Scientist, Physics Departments, Boğaziçi and Koç Universities, Istanbul, Turkey, 1-5 November 2010.
• Visiting Scientist, Physics Department, Mons University, Belgium, 25 October 2010.
• Seminar, "Potentials Unbounded Below", Centro de ciencias de Benasque Pedro Pascual, Spain, 29 July 2010.
• Participant, *Supersymmetric Quantum Mechanics and Spectral Design*, Centro de ciencias de Benasque Pedro Pascual, 25-30 July 2010.
• Participant, *Non-Perturbative Techniques in Field Theory*, London Mathematical Society - EPSRC Durham Symposium, 18-25 July 2010.
• Participant, *Planck 2010: From the Planck Scale to the ElectroWeak Scale*, CERN, 31 May - 4 June 2010.
• Co-organizer and seminar, "Associativity, Jacobi, Bremner, and All That", *Miami 2009*, 15-20 Dec 2009.

Professor Mezincescu's recent activities:

•Member of the International Advisory Board of the International Conference- Quantum Field Theory and Gravity, Centre of Theoretical Physics Tomsk State Pedagogical Institute, Tomsk Russian Federation July 28 - August 3 2014.
• Invited participant *From the Renormalization Group to Quantum Gravity: Celebrating the science of Joe Polchinski*, February 27 - February 28, 2014 at KITP Santa Barbara California.
• Invited Participant *Workshop: Aspects of Supergravity* January 6-10, 2014 at the Simons Center for Geometry and Physics., SUNY Stony Brook New York.

• Member in the Steering Committee and participant, *Miami 2012*, 12-18 Dec 2013, Fort Lauderdale Florida.

• Member of the International Advisory Board of the International Workshop Supersymmetries and Quantum Symmetries (SQS' 2013) JINR- Dubna, Russian Federation, July 29- August 3 2013

• Invited participant of *Workshop on String Theory*, Centro de Ciencias de Benasque Pedro Pasqual, Benasque Spain June 30- July 12, 2013.

• Invited participant, *Workshop: Adventures in Superspace*, 19-20 Apr 2013, McGill University, Montreal, Canada.

• Member in the Steering Committee and participant, *Miami 2012*, 13-20 Dec 2012, Fort Lauderdale Florida.

• Invited participant and talk: *Hamiltonian Formulation of Open Super(Strings), Branes, Supergravity and M-Theory* A conference to celebrate the 60th birthday of Paul K Townsend, DAMTP-University of Cambridge 2nd - 3rd July, 2012, Cambridge, UK.

• Visiting Fellow, *Mathematics and Applications of Branes in String and M-theory*, Isaac Newton Institute for Mathematical Sciences, Cambridge, UK, 16-31 May 2012.

• Co-organizer and participant, *Miami 2011*, Fort Lauderdale, Florida, 15-20 Dec 2011.

•Invited Participant at Workshop on String Theory, Benasque, Spain, 18-30 July 2010.

• Co-organizer and participant, *Miami 2010*, Fort Lauderdale, Florida, 14-19 Dec 2010.

•Invited talk "Anyons, Strings and Superstrings", at Workshop on Supersymmetric Quantum Mechanics and Spectral Design, Benasque, Spain, 3-15 Jul 2010.

• Invited talk "Anyons and Strings", at International Conference Quantum Field Theory and Gravity, Tomsk, Russia, 5-9 Jul 2010.

• Invited Short Term Visitor, DAMTP-University of Cambridge, Cambridge, UK, Mar 16 - Apr 25, 2010.

• Invited Short Term Visitor, Weizmann Institute of Science, Rehovot, Israel, Jan 7 - Feb 7, 2010.

• Invited talk "Super Landau Models on Some Supermanifolds", at Joint Seminars in Theoretical High Energy Physics Hebrew University, Tel-Aviv University, Weizmann Institute at Neve Shalom/Wahat al-Salam, Jan 19, 2010.

•Member of the International Advisory Board of the International Conference- Quantum Field Theory and Gravity, Centre of Theoretical Physics Tomsk State Pedagogical Institute, Tomsk, Russia, 5-9 Jul 2010.

• Member of the Advisory committee,and participant of the Workshop Supersymmetric Quantum Mechanics and Spectral Design, Centro de Ciencias de Benasque Pedro Pasqual, Benasque, Spain, 18-30 Jul 2010.

• Co-organizer, participant, and invited talk, "Super Landau Models, " *Miami 2009*, Fort Lauderdale, Florida, 15-20 Dec 2009.

Recent publications

Complete lists of publications for the PI and CoPI may be found on InSPIRE. Here is a list of the most recent papers (last five years).

• T. Curtright and T.S. Van Kortryk, "On Rotations as Spin Matrix Polynomials" to appear in J. Phys. A: Math. Theor., arXiv:1408.0767 [math-ph]

• T. Curtright, D. B. Fairlie and C. K. Zachos, "A compact formula for rotations as spin matrix polynomials " SIGMA 10 (2014) 084, arXiv:1402.3541 [math-ph]

• L. Mezincescu, A. J. Routh and P. K. Townsend, "All Superparticles are BPS" J. Phys. A 47 (2014) 175401, arXiv:1401.5116 [hep-th]

• T. Curtright, D. B. Fairlie and C. K. Zachos, *A Concise Treatise on Quantum Mechanics in Phase Space*, World Scientific and Imperial College Press, ISBN: 978-981-4520-43-0.
• L. Mezincescu, A. J. Routh and P. K. Townsend, "Supertwistors and massive particles" Annals Phys. 346 (2014) 66, arXiv:1312.2768 [hep-th]
• T. Curtright and C. K. Zachos, "Branched Hamiltonians and Supersymmetry" J. Phys. A: Math. Theor. 47 (2014) 145201, arXiv:1311.6147 [math-ph]
• L. Mezincescu, A. J. Routh and P. K. Townsend, "Equivalence of 3D Spinning String and Superstring" JHEP 1307 (2013) 024, arXiv:1305.5049 [hep-th]
• T. Curtright and C. K. Zachos, "Umbral Vade Mecum" Front. Physics 1 (2013) 15, arXiv:1304.0429 [math-ph]
• T. Curtright and D. B. Fairlie, "A Galileon Primer" arXiv:1212.6972 [hep-th]
• T. Curtright,"Galileons and Naked Singularities" Phys. Lett. B716 (2012) 366-369, arXiv:1208.1205 [hep-th]
• T. Curtright and D. B. Fairlie,"Geons of Galileons" Phys. Lett. B716 (2012) 356-360, arXiv:1206.3616 [hep-th]
• L. Mezincescu and P. K. Townsend, "3D strings and other anyonic things" Fortsch. Phys. 60 (2012) 1076, arXiv:1111.3384 [hep-th]
• T. Curtright, X. Jin, and C. Zachos, "RG flows, cycles, and c-theorem folklore" Phys. Rev. Lett. 108 (2012) 131601, arXiv:1111.2649 [hep-th].
• L. Mezincescu and P. K. Townsend, "Quantum 3D Superstrings" Phys. Rev. D84, 106006 (2011), arXiv:1106.1374 [hep-th]
• T. Curtright, X. Jin, and C. Zachos, "Approximate Solutions of Functional Equations" J. Phys. A: Math. Theor. 44 (2011) 405205, arXiv:1105.3664 [math-ph].
• T. Curtright and C. Zachos, "*Quantum Mechanics in Phase Space*" Asia Pacific Newsletter 1 (2012) 36-45, arXiv:1104.5269 [physics.hist-ph].
• T. Curtright, "Potentials Unbounded Below" SIGMA 7 (2011) 042, arXiv:1011.6056 [math-ph].
• L. Mezincescu and P. K. Townsend, "The Quantum 3D Superparticle" SIGMA **7**, 005 (2011), arXiv:1011.5049 [hep-th].
• T. Curtright and C. Zachos, "Renormalization Group Functional Equations" Phys. Rev. D83 (2011) 065019, arXiv:1010.5174 [hep-th].
• L. Mezincescu and P. K. Townsend, "Semionic Supersymmetric Solitons" J. Phys. A 43, 465401 (2010), arXiv:1008.2775 [hep-th].
• L. Mezincescu and P. K. Townsend, "Anyons from Strings" Phys. Rev. Lett. 105, 191601 (2010), arXiv:1008.2334 [hep-th].
• T. Curtright, "Strings on a plane" Phys. Lett. B693 (2010) 477-480.
• T. Curtright and A. Veitia, "Logistic Map Potentials" Phys.Lett. A375 (2011) 276-282, arXiv:1005.5030 [math-ph].
• T. Curtright, "Associativity, Jacobi, Bremner, and All That", to appear in Journal of Physics, arXiv:1003.4258 [hep-th].
• A. Beylin, T. Curtright, E. Ivanov, and L. Mezincescu, "Generalized $N = 2$ Super Landau Models" JHEP 1004:091 (2010), arXiv:1003.0218 [hep-th].
• T. Curtright and C. Zachos, "Chaotic Maps, Hamiltonian Flows, and Holographic Methods" J. Phys. A: Math. Theor. 43 (2010) 445101, arXiv:1002.0104 [nlin.CD].
• T. Curtright and C. Zachos, "Evolution profiles and functional equations" J. Phys. A: Math. Theor. 42 (2009) 485208, arXiv:0909.2424 [math-ph].
• T. Curtright, X. Jin, and L. Mezincescu, "Multi-operator brackets acting thrice" J. Phys. A: Math. Theor. 42 (2009) 462001, arXiv:0905.2759 [math-ph].

• T. Curtright, D. Fairlie, X. Jin, L. Mezincescu, and C. Zachos, "Classical and Quantal Ternary Algebras" Phys. Lett. B675 (2009) 387-392, arXiv:0903.4889 [hep-th]
• L. Mezincescu "Quantum Mechanics on some Supermanifolds ", to appear in *Quantum Mechanics of Fundamental Systems: The Quest for Beauty and Simplicity Claudio Bunster Festschrift* M. Henneaux and J. Zanelli, editors, Springer 2009.

Accomplishments of prior results with intellectual merit

Some examples of branched Hamiltonians were explored both classically and in the context of quantum mechanics, as recently advocated by Shapere and Wilczek. These are in fact cases of switchback potentials, albeit in momentum space, as previously analyzed by us for quasi-Hamiltonian chaotic dynamical systems in a classical setting, and as encountered in analogous renormalization group flows for quantum theories which exhibit RG cycles. A basic two-worlds model, with a pair of Hamiltonian branches related by supersymmetry, was considered in detail.

A preliminary version of our Galileon Primer, wherein elementary features of galileon models are discussed at an introductory level, was posted on the arXiv. A revised version is in preparation and nearly complete. The revision updates the references and places additional emphasis on Legendre duality. Additional implicit and explicit solutions are constructed and analyzed in some detail.

The (super) twistor formulation, in D=3 and 4, and 6 space-time dimensions, of the dynamics of various massless and massive spinning particles and superparticles, was developed and reviewed. In this formulation the (super)twistor variables are alternatives to the usual phase–space and do not append the standard variables as in most of the alternative approaches. An explicit proof was given of how in the 6D case, charges corresponding to an internal SU(2) gauge invariance of the twistor formulation are related to the particle spin. This was established by showing that the SU(2) triplet constraints are related to the 6D generalization of the Pauli-Lubanski spin vector. A similar relation was shown to hold in the case of massive 4D particle mechanics. The extension to D=4 of the 3D results in the literature that the N=1 massive superparticle has N=2 supersymmetry and that it is equivalent to the N=2 BPS superparticle was also proved. It was further shown that the N-extended massive superparticle action in 4D has 2N-extended supersymmetry. The generic action for an N-extended superparticle in D-dimensional Minkowski spacetime was also shown to have 'hidden' supersymmetries (related by 'dualities' to the manifest supersymmetries) such that the full supersymmetry algebra is BPS-saturated; the exceptions are those for which the manifest supersymmetry algebrais already BPS-saturated. Moreover, it was shown that any 'non-BPS' superparticle action is a gauge-fixed version of the 'BPS' superparticle action for which all supersymmetries are manifest.

Prior results having broader impact

There are many ways in which our results have contributed to education and overall intellectual advancement, thereby increasing the economic competitiveness of this country. (This is described in more general terms at the end of this proposal.) Here, however, we wish to point out specific results produced while supported by our previous NSF award. The textbook *A Concise Treatise on Quantum Mechanics in Phase Space*, listed above, is one such example. It is the direct result of our research on the subject, but it is not a research document. Rather, it is designed to improve how quantum mechanics is taught and appreciated in university classes, across all disciplines, as carefully explained in the book. We believe it succeeds to achieve this goal. The resulting impact could be *very* broad. Moreover, it covers some little known historical background, echoing remarks made in the above Asia Pacific Newsletter article, and intended to appeal to a general readership.

At a slightly more technical level, the two articles listed on spin matrix expansions are also intended for a general, multidisciplinary audience. These articles address and solve elementary issues involving spin in quantum mechanics, issues left unsettled since the work of Wigner in the 1930s, even though those issues have been tackled several times, but without being fully resolved, during the intervening 80 years. Our work finally resolves these issues. This too should have broad impact.

Proposed research

The work proposed here is largely a continuation of our previously funded research in theoretical particle physics. Often that previous work has led to unanticipated yet very interesting new paths to explore. In all, we believe the proposed research pushes the envelope of physics as well as the reach of science.

Branched Hamiltonian Systems

We propose to continue to explore examples of branched Hamiltonians, both classically and in the context of quantum mechanics, as recently advocated by Shapere and Wilczek [67, 68].

Multi-valued Hamiltonians have appeared in at least two contexts. Most recently, they have resulted from Legendre transforming Lagrangians whose velocity dependence is not convex [67, 68], which invariably leads to a Riemann surface phase-space structure, with multiply-branched Hamiltonians, and to interesting topological issues [69, 79]. But, previously, they have arisen in the continuous interpolation of discrete time dynamical systems, particularly those systems that exhibited chaotic behavior, where they could be incorporated in a canonical "quasi-Hamiltonian" formalism [30, 31, 29, 24]. (Moreover, by analogy with quasi-Hamiltonian systems, renormalization group flows that exhibit cycles have also been shown to be governed by multi-valued β functions [33, 28].)

In our previous work [32] we considered several simple Lagrangian models that lead to double-valued Hamiltonian systems, to illuminate "two-worlds theory." We began with an example where the velocity dependence of L is given by a gaussian. This example illustrates many generic features of branched Hamiltonians, in addition to its more specific peculiarities. In particular, as a quantum system the gaussian model is not amenable to solution in closed form, so we turned to a different class of models where analytic results can be obtained. One of the models in this class was tailored so as to have a pair of Hamiltonians that comprise a supersymmetric quantum mechanical system [80]. This facilitated obtaining analytic results as well as a numerical study of this special model.

The supersymmetric system in question is given by a cube root Lagrangian, and a corresponding double-valued Hamiltonian.

$$L = C\,(v-1)^{1/3} - V\,(x) \underset{\text{Legendre}}{\Longleftrightarrow} H_\pm = p \pm \frac{1}{2\sqrt{p}} + V\,(x)\ , \quad V\,(x) = x^2 \underset{\text{QM in } p \text{ space}}{\longmapsto} -\frac{d^2}{dp^2}\ . \tag{1}$$

The *momentum space* pair of QM Hamiltonian operators for this case is expressible in the standard form for a supersymmetric pair,

$$H_\pm = -\frac{d^2}{dp^2} + w_0^2\,(p) \pm w_0'\,(p) = \left(\frac{d}{dp} \pm w_0\,(p)\right)\left(-\frac{d}{dp} \pm w_0\,(p)\right)\ , \tag{2}$$

where $w_0\,(p) = \sqrt{p}$. For hermiticity, there is a restriction $p \geq 0$. The model has the interesting feature that the true — square-integrable — ground state of the system is non-vanishing for only one of the branches, namely, H_-.

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The fact that the multi-valued Hamiltonian has but two branches allows for the features of the model to conform with well-known expectations for general supersymmetric QM. Due to the restriction $p \geq 0$, there is perhaps an interesting wrinkle here, albeit previously encountered for the supersymmetric simple harmonic oscillator (but normally expressed in terms of $\psi(x)$): The degenerate $H_{\pm}$ eigenfunctions obey different boundary conditions at $p = 0$. If one is Dirichlet, the other is Neumann. This follows from the mutual relations between $\psi_E^{(\pm)}$ and the effects of d/dp when acting on nonsingular functions at $p = 0$. For example, the first H_- excited state and its degenerate H_+ partner eigenstate satisfy $\psi_{E_1}^{(-)}\Big|_{p=0} = 0 = d\psi_{E_1}^{(+)}/dp\Big|_{p=0}$, while for the next excited states, $d\psi_{E_2}^{(-)}/dp\Big|_{p=0} = 0 = \psi_{E_2}^{(+)}\Big|_{p=0}$, etc. Flipping the boundary conditions actually has a practical benefit due to the $1/\sqrt{p}$ singularity in both $H_{\pm}$: It is more straightforward to perform an accurate numerical computation of the energy eigenvalue using the boundary condition $\psi_E(0) = 0 \neq \psi'_E(0)$ than it is using the condition $\psi_E(0) \neq 0 = \psi'_E(0)$. The degeneracy of the eigenfunctions permits one to always choose the $\psi_E(0) = 0$ condition, along with the corresponding H_+ or H_-.

These higher energy states may be thought of as a single nontrivial state defined on a unified covering space — a double covering of the half-line $\mathbb{R}_+$ by $\mathbb{R}$ — obtained by unfolding the two Hamiltonian branches to obtain a single H [68] globally defined on $\mathbb{R}$. However, the true ground state of the system is $\psi_0(p) \cup 0$ on the unfolded space. The latter, somewhat unusual feature is possible because the two Hamiltonians on the half-lines join together in a cusp at $p = \infty$, where ψ_0 and all its derivatives vanish. So too vanish all the higher $\psi_E^{(\pm)}$ and all their derivatives at $p = \infty$. For this reason, it would be excusable not to have thought of the degenerate eigenstates on the half-line as two branches of a single function. However, the unified two-worlds picture provided by joining them together on a covering real line, with Neumann and Dirichlet boundary conditions at opposite ends, is a more compelling point of view, in our opinion. Perhaps more importantly, this omniscient view of the two-worlds system becomes natural when the common Lagrangian underpinning both $H_{\pm}$ is considered.

In this project we propose to search for and classify field theories that have branched Hamiltonians or related structure. This is largely unexplored territory. But there are a class of field theories where Legendre transformations naturally come into play. These are galileon models. Thus we propose to consider these models in a branched Hamiltonian framework.

Galileons

Galileon theories are a class of models for new scalar fields whose Lagrangians involve multilinears of first and second derivatives, but whose nonlinear field equations are nonetheless still only second order. They may be important for the description of large-scale features in astrophysics as well as for elementary particle theory [37, 43]. Hierarchies of such Lagrangians giving rise to such field equations were first discussed mathematically in [53, 45, 46, 47]. The simplest example involves a single scalar field. This galileon field is usually coupled to all *other* matter through the trace of the energy-momentum tensor, $\Theta^{(\text{matter})}$, and is thus gravitation-like by virtue of the similarity between this universal coupling and that of the metric $g_{\mu\nu}$ to $\Theta_{\mu\nu}^{(\text{matter})}$ in general relativity. But for a self-consistent theory, for the coupling to be truly universal, the galileon should also be coupled to its own energy-momentum trace, even in the flat spacetime limit. Some consequences of this additional self-coupling are discussed in our papers [24, 25, 26].

The action for the lowest non-trivial member of the galileon hierarchy can be written in various ways upon integrating by parts. Perhaps the most elegant of these is

$$A_2 = \tfrac{1}{2} \int \phi_\alpha \phi_\alpha \phi_{\beta\beta} \; d^n x \ . \tag{3}$$

where ϕ is the scalar galileon field, $\phi_\alpha = \partial\phi(x)/\partial x^\alpha$, etc., and where repeated indices are summed using the Lorentz metric $\delta_{\mu\nu} = \text{diag}(1, -1, -1, \cdots)$. The field equation that follows from A_2 is $0 = \mathcal{E}_2[\phi]$, where $\mathcal{E}_2[\phi] \equiv \phi_{\alpha\alpha}\phi_{\beta\beta} - \phi_{\alpha\beta}\phi_{\alpha\beta}$. This is second order, albeit nonlinear. The A_2 term contributes a nonzero trace to the energy-momentum tensor for the galileon field. It is then natural also to self-couple the field to this trace. Therefore, we considered a model with action

$$A = \int \left(\tfrac{1}{2}\phi_\alpha\phi_\alpha - \tfrac{1}{2}\lambda\phi_\alpha\phi_\alpha\phi_{\beta\beta} - \tfrac{1}{4}\kappa\phi_\alpha\phi_\alpha\phi_\beta\phi_\beta \right) \; d^4x \ , \tag{4}$$

where for the Lagrangian we took a mixture of three terms: the standard bilinear, the trilinear galileon, and its corresponding quadrilinear trace-coupling. The quadrilinear is reminiscent of the Skyrme term in nonlinear σ models [71] although here the topology would appear to be always trivial. The second and third terms in A are logically connected, as we have indicated. But why include in A the standard bilinear term? The reasons for including this term are to soften the behavior of solutions at large distances, and also to satisfy Derrick's criterion for classical stability under the rescaling of x. Without the bilinear term in L the energy within a spatial volume would be neutrally stable under a uniform rescaling of x, and therefore able to disperse [40, 42].

Following the lead of many other studies, we discussed classical solutions of the model (4) in our papers [24, 25, 26]. We then considered minimal covariant coupling of the model to standard gravity. We discovered novel general relativistic effects. We were surprised to discover the model readily exhibits *naked singularities*, unlike the usual situation where only a bilinear scalar field action is coupled minimally to gravity. With the static metric expressed in Schwarzschild coordinates as

$$(ds)^2 = e^{N(r)}(dt)^2 - e^{L(r)}(dr)^2 - r^2(d\theta)^2 - r^2\sin^2\theta\,(d\varphi)^2 \ , \tag{5}$$

we define the constants M and C in the usual way in terms of $r \to \infty$ data for static solutions in asymptotically flat spacetime:

$$e^{L/2} \underset{r\to\infty}{\sim} 1 + \frac{M}{r} + O\left(\frac{1}{r^2}\right) \ , \quad e^{N/2} \underset{r\to\infty}{\sim} 1 - \frac{M}{r} + O\left(\frac{1}{r^2}\right) \ , \quad \phi \underset{r\to\infty}{\sim} -\frac{C}{r} + O\left(\frac{1}{r^2}\right) \ . \tag{6}$$

Based on a numerical study, we found solutions with naked singularities separated from those with event horizons by a critical curve in the space of asymptotic data (i.e. the (M, C) plane) such that the data space is nearly equipartitioned. So, in this model, naked singularities are common occurrences [26].

In this project, we propose to extend these results in several ways. First, it would be interesting to consider how generic initial data evolves to form either a naked singularity or an event horizon for the specific model at hand. This would require extensive numerical work along the lines developed by Choptuik and his collaborators [17, 18]. We are working to develop a collaboration with other relativists to study this question. Second, the model can be investigated in either lower or higher spacetime dimensions. Some other models in three dimensions are known to permit analytic results to be obtained [3, 77]. Perhaps this is also true for galileons coupled to gravity in 3D. In higher dimensions, there are singularities with nonspherical topologies (e.g. black rings [51, 56, 57]). It would be interesting to determine the effects of a galileon scalar on such configurations. The obvious conjecture would be that, again, the phase space of initial data is partitioned into distinct regions, one giving rise to naked singularities and one producing event horizons, with a critical boundary separating the two. We propose to study these questions as part of this project.

Legendre Duality for Galileons

The proceeding two research topics naturally come together for galileon models. The various Lagrangians in the galileon hierarchy are interchanged by spacetime Legendre transformations [26, 38, 39]. The standard form for a *Legendre transformation* relating fields and spacetime variables, $\phi, x \longleftrightarrow \Phi, X$, is given by

$$\phi(x) + \Phi(X) = \sum_{\alpha=1}^{n} x_\alpha X_\alpha \ , \qquad X_\alpha(x) = \frac{\partial \phi(x)}{\partial x_\alpha} \equiv \partial_\alpha \phi \ , \qquad x_\alpha(X) = \frac{\partial \Phi(X)}{\partial X_\alpha} \equiv \nabla_\alpha \Phi \ . \tag{7}$$

It follows that the Hessian matrices for ϕ and Φ, if nonsingular, are related by $(\partial\partial\phi)^{-1} = (\nabla\nabla\Phi)$. From this it follows in n dimensions that $\frac{1}{\sqrt{\det(\partial\partial\phi)}} \frac{1}{k!} \mathcal{E}_k(\partial\partial\phi) = \frac{1}{\sqrt{\det(\nabla\nabla\Phi)}} \frac{1}{(n-k)!} \mathcal{E}_{n-k}(\nabla\nabla\Phi)$. That is to say, field equations for Galileons ϕ and Φ are related by the Legendre tranform. Thus the transformation gives a one-to-one local map between solutions of the nonlinear equations. This provides a general, implicit procedure for the construction of solutions to the equation $\mathcal{E}_k = 0$ given solutions to $\mathcal{E}_{n-k} = 0$.

The Legendre transformation also provides a duality relation between actions:

$$\frac{1}{k!} \mathcal{A}_k[\phi] = \frac{(-1)^n}{(n-k)!} \mathcal{A}_{n-k}[\Phi] \tag{8}$$

for Lorentzian spacetimes. In principle the quantum theories for k and $n-k$ are therefore related in n spacetime dimensions. This has been verified by explicit perturbative calculations about flat spacetimes [38, 39].

But integrations by parts are needed to show (8), and boundary terms have been dropped. This is not always allowed. Moreover, Legendre transformations are multi-valued, in general, as exemplified by the theory of branched Hamiltonians. The relations (7) may therefore *fold the spacetime manifold* in such multi-valued cases, converting infinite expanses into finite regions with nontrivial boundaries, and vice versa. Quantum effects can be very exotic in such situations. We propose to investigate these phenomena as part of this project.

3D Spinning Strings and Superstrings

There are two standard formulations of the ten-dimensional (i.e. critical dimension) superstring theory. The first to be found was the "RNS formulation" which was obtained, as a free string theory, by removing from the combined spectrum of the Ramond [175] and Neveu-Schwarz [176] spinning strings the states which do not form multiplets of spacetime supersymmetry using the Gliozzi-Scherk-Olive (GSO) projection [177]. An alternative light-cone gauge action with manifest spacetime supersymmetry was then proposed by Green and Schwarz [178] and shown by them to be equivalent (by virtue of the triality property of the Spin(8) transverse rotation group) to the light-cone gauge-fixed RNS superstring; in this alternative "GS formulation" the GSO projection is transformed into the simple requirement of Ramond-type boundary conditions on the fermionic variables (i.e. periodicity for a closed string). Green and Schwarz subsequently found the covariant form of their alternative string action [179], and this is the natural starting point of the GS formulation of superstring theory.

An intriguing feature of the GS formulation is that even the classical superstring action exists only for spacetime dimensions $D = 3, 4, 6, 10$. As for the RNS string, quantization of the light-cone gauge-fixed action preserves Lorentz invariance if $D = 10$ but not otherwise for $D \geq 4$. Recently, it was shown that the Lorentz invariance, and spacetime supersymmetry, of the superstring are also preserved when $D = 3$ although the spectrum then contains particles of *irrational* spin [117]. It is not known how to recover this result by covariant quantization of the 3D GS superstring; one difficulty is that the covariant wave equation for a particle of irrational spin requires an infinite-component wave-function [180, 181].

Apart from this difficulty there are other well-known difficulties in the covariant quantization of the superstring in the GS formulation, so it could be useful if there were some 3D analog of the critical dimension equivalence with the RNS superstring. In [192] it is given a proof that the 3D GS closed superstring with $\mathcal{N} = 2$ space-time supersymmetry is equivalent to the Ramond-Ramond sector of the 3D closed spinning string, which is referred as the (3D) "Ramond string". In other words, there is indeed a 3D analog of the critical dimension GSO projection: it involves projecting out the sectors involving NS boundary conditions. This result is established by performing a light-cone gauge quantization of the 3D Ramond string and comparing to the analogous GS superstring results obtained previously [117]. The spectrum is found to be identical. This means, in particular, that there are states of irrational spin in the spectrum of the 3D spinning string.

A corollary of this result is that the worldsheet supersymmetric 3D Ramond string actually has 3D space-time supersymmetry. The equivalence mentioned to the 3D Green-Schwarz superstring is then used, to construct the space-time supersymmetry charges of the 3D Ramond string, in light-cone gauge. We are in the process of extending these results to various open spinning and super strings.

All Superparticles are BPS

[193] is concerned, principally, with the massive superparticle. In some respects this is simpler than the massless superparticle because, generically, massive superparticle actions do not have fermionic gauge invariances. However, there is a close connection between the massless and massive cases. The results are valid for a general spacetime dimension but because properties of spinors are dimension dependent it is not convenient to consider all dimensions at once. For simplicity of presentation, it was assumed that $D = 3, 4$ or 10 (mod 8) in which case one may assume that the spinor coordinate Θ of $N = 1$ superspace is Majorana, with a Majorana conjugate, $\bar{\Theta} = \Theta^T C$, where the charge conjugation matrix C is antisymmetric, and also that Θ is chiral or anti-chiral if $D = 10$. The action, in Hamiltonian form, for the $N = 1$ superparticle of mass m is

$$S = \int dt \left\{ \dot{X} \cdot P + i\bar{\Theta} \not{P} \dot{\Theta} - \frac{1}{2} e \left(P^2 + m^2 \right) \right\} , \tag{9}$$

where Γ are the Dirac matrices, and an overdot indicates a derivative with respect to the arbitrary worldline coordinate t. The "einbein" e is a Lagrange multiplier for the mass-shell constraint. For the explanation of the rest of the notations one can consult [193].

[193] detaillates two observations about massive superparticle actions, such as (9). The first is that they have additional "hidden" (i.e. non-manifest) supersymmetries, such that the full supersymmetry algebra is the same as that of a higher-dimensional massless superparticle that has been dimensionally reduced by fixing the components of the momentum in the extra dimension(s), which then appear in the full lower-dimensional supersymmetry algebra as central charges. Unitarity implies a bound on the mass in terms of the central charges, which is referred to as the BPS bound, and the construction described leads to a supersymmetry algebra in which this bound is saturated. In this sense, all massive superparticles "are BPS", as are (trivially) massless superparticles.

The hidden supersymmetries of massive superparticle actions are related to the manifest ones by a "duality". In the $D = 3, 4$ cases this is a self-duality in the sense that the massive superparticle action is mapped into itself; this implies a $\mathbb{Z}_2$ automorphism of the full supersymmetry algebra.

The second observation in [193], is that this equivalence holds not just for $D = 3, 4$, but for *any* dimension. The $N = 1$ massive superparticle is just a gauge-fixed version of the $N = 2$ BPS superparticle, which explains why the former has "hidden" supersymmetries. This by itself is not sufficient for equivalence because a gauge fixing that breaks manifest Lorentz invariance would need to be followed by a field redefinition that restores it, and then there is no guarantee that the Lorentz Noether charges of the $N = 2$ BPS superparticle will coincide with those of the massive $N = 1$ superparticle. However, it is possible to fix the kappa-symmetry Lorentz-covariantly; this is not possible for a massless superparticle but it becomes possible upon dimensional reduction, i.e. for a BPS superparticle, because the relevant Lorentz group is then that of the lower dimension. When the kappa-symmetry is gauge fixed in this way, the $N = 2$ BPS action becomes equivalent to the massive $N = 1$ action. We plan to further investigate the consequences of the above statements.

Prospectus of noncommutative geometry and coset research

Some of our previous results demand further work in order to deepen their conceptual impact. For example the emergence [81, 9] of sound quantum systems corresponding to particles moving on supermanifolds and of new $\mathcal{N} = 2$ quantum mechanical SUSY models [10] is certainly a direction worth pursuing. We are dealing here with a completely different mechanism for implementation of a symmetry and such a mechanism cannot be overlooked or ignored. There is a huge number of models which can be constructed by techniques similar to those pursued by us. However, these models are waiting for their physics context, yet to be found.

Another issue which should be addressed is that of the representations content of the $SU(2|2)$ symmetry uncovered for the superflag Landau model [9], within the study of quantum mechanics on super manifolds. Preliminary results indicate these are new types of short representations previously unknown, and we hope that this may be of certain use in other contexts [6]. It should be stressed that we have a very unconventional construction of the generators of $SU(2|2)$, connected with the positivity of the norm, which may uncover some subtle properties.

An interesting general issue, not yet investigated, is how the semi-classical limit is modified by a change in the Hilbert space metric. In the coherent state approach to the classical limit, the symplectic 2-form associated to the classical dynamics clearly depends on the Hilbert space metric. A change from a non-positive metric to a positive one cannot be unitary, so we should expect a non-canonical transformation of the classical phase space. However, the negative norms that we find for the 'naive' Hilbert space metric are associated with the anti-commuting variables for which there is no truly classical limit, but nevertheless being able to find a trace of the full quantum action at the level of the formal semiclassical actions involving anticommuting variables maybe of great interest especially for uncovering their mysterious dynamics.

There remains also the issue of using the positive definite Hilbert space metric approach for supersymmetric relativistic models. That is constructing superparticle actions of Brink-Schwarz type which poses a non trivial Hilbert space metric, compatible with the Lorentz invariance and SUSY. This can be accomplished by considering superparticle actions with higher derivatives [140] on the spinor variables which typically lead to a spectrum which contains different supermultiplets of different signatures for which it should be possible to recover only positive signatures. Ultimately one would like to know whether one can generate new types of strings or membranes by this approach.

We possess models in non-trivial backgrounds which exhibit non trivial alternative norms, however the ultimate validity of such an approach will be in the success of constructing second quantized theories which exhibit such invariances. This is potentially a very interesting major direction whose possibility of development may be quite significant.

We also intend to consider other problems related to the super-sphere. There are some investigations on the possibility that there may be some ambiguity in its definition and that pursue the use of the so-called star adjoint operation to construct simple physical models [101]. Further outgrowths of geometric quantization on super-manifolds will be the quantum equations of certain 'fuzzy' $CP^{(n|m)}$. In fact thereis an anticipated, very deep relation, still to be determined, between the classical geometry of co-adjoint orbits of supergroups such as $SU(n|m)$ and the representation content of the Hilbert superspaces of quantum particles on these spaces. The point is that the representations which occur are always "atypical", and this must be reflected in the classical geometry at the semi-classical limit.

Supertwistors and massive particles

Twistors were introduced as a means of understanding massless free field equations, which are what one finds by quantization of a model of massless particle mechanics. The motivation for twistors in this context is partly that they make the conformal invariance of massless particles manifest. It is therefore rather surprising that twistor methods are still applicable when the conformal invariance is broken by the inclusion of a mass [158]. To some extent this is explained by the massless higher-dimensional origin of some models of massive particle mechanics. In any case, the motivation for a twistor reformulation of *massive* particle mechanics is no longer that it makes manifest otherwise non-manifest symmetries. Instead, the motivation (at least for our purposes) comes largely from the simplifications it brings to the mechanics of massive particles with spin, where the spin is introduced by the inclusion of anticommuting phase space coordinates.

In [191] the twistor formulation of the mechanics of massless particles in a Minkowski spacetime of three or four dimensions was extended to the massive particle case. In the three-dimensional (3D) case, a distinguishing feature of the formalism developed is that the twistor variables are used to replace the usual phase space variables rather than augment them. This is in the spirit of Shirafuji's approach to four-dimensional (4D) massless particle mechanics [148], which yields particular 3D examples of massive particle mechanics upon dimensional reduction; i.e. upon setting a component of the 4-momentum equal to a constant mass. In this process, a 4D twistor variable is replaced by a pair of 3D twistor variables [160], but the resulting bi-twistor formalism is generally applicable and is not restricted to those cases which have a 4D origin.

It is also worth mentioning the elucidation of the physical significance of the spin-shell constraint functions, which generate a local $SU(2)$ gauge invariance of the action. In the massless 6D case the spin-shell constraint functions are 6D analogs of 4D helicity; in the massive 4D case, the quadratic Casimir of the $SU(2)$ gauge group, which is the sum of squares of the constraint functions, is proportional to the square of the Pauli-Lubanski spin-vector.

There is a supertwistor formulation of massless superparticle mechanics in space-time dimensions $D = 4$ and 6, and this leads to a bi-supertwistor formulation of some models of massive superparticle mechanics in $D = 3, 4$. Again, this bi-supertwistor formalism is generally applicable. An advantage of the (super)twistor formulation in this context is that the spin content can be read off from the spin-shell constraints. Essentially, particle helicities are given by a formula involving fermion occupation numbers which take only the values 0 and 1, yielding a finite supermultiplet.

A general feature of the models considered here is that spin is incorporated through the introduction of anticommuting variables. In the supertwistor formulation this amounts to the inclusion of some number of fermi oscillators with corresponding fermi number operators appearing in the spin-shell constraint, possibly subject to constraints that relate them. Different models in the same dimension and with the same particle mass differ only in the number of fermi oscillators, how they appear in the spin-shell constraint, and whether there are relations between them. In the 3D case, it is possible to introduce spin without the need for anticommuting variables via the introduction of a Lorentz-Wess-Zumino term (first discussed in [167] although the terminology used here was introduced in [173]). This possibility is particularly transparent in the (super)twistor formulation; it just amounts to the addition of a constant to the spin-shell constraint.

The above mentioned results have been used in [194] to attempt a massive bi twistor 4D formulation for massive particles with spin with the use of the so called Souriau form written in terms of the spin four vector. In [195] the bi-twistor approach was used to describe an infinite dimensional multiplet of massive higher spin particles. We plan to analyze such issues in our approach.

We also plan to analyze a possible intriguing connection between the twistor formalism for massive and massless particles and super particles treated by the co-adjoint orbit method in the spirit of [196] and [197]. One deals with two different ways of solving the mass shell constraint and it is natural to assume that they are related in some way. The co-adjoint orbit method has been lately of interest in connection with relativistic fluids and magnetohydrodynamics with spin [198] and anomalous velocity terms for particles with spin coupled to gravitational fields [199, 200].

Finally as in strain theory there appear infinite towers of massive particles it is natural to ask about its twistor formulation. Although the existence of a Lorentz invariant quantum 3D string can be seen in the light-cone gauge, standard methods of covariant quantization appear to exclude it. This appears to be related to the role of the conformal anomaly, which arises either as an anomaly in the residual gauge invariance in conformal gauge (e.g. in path-integral quantization) or more directly as an anomaly in the algebra of the Hamiltonian constraints. In the latter case, the anomaly implies that the constraints can not all be imposed as physical-state conditions. This difficulty is bypassed in light-cone gauge because this gauge allows the Hamiltonian constraints to be solved prior to quantization. The conformal anomaly is still relevant, however, because Lorentz transformations must now be accompanied by compensating conformal transformations, so a conformal anomaly can now manifest itself as a Lorentz anomaly. It does so for all $D > 3$ except $D = 26$ (because this is the critical dimension in which the conformal anomaly cancels) but $D = 3$ is another exception [83, 117](at least for the free string, which is all that we consider here). In other words, whereas the critical dimension string is Lorentz invariant, as a quantum theory, because there is no conformal anomaly, the 3D string is Lorentz invariant *despite* the conformal anomaly.

It is natural to suppose that there must be a way to recover this light-cone gauge result by some manifestly Lorentz covariant quantization procedure, one in which the conformal anomaly plays a role that is more analogous to its role in light-cone gauge quantization than to its role in standard covariant quantization procedures. This might be achieved if the Hamiltonian constraints could be solved *without breaking Lorentz invariance*. We plan to deal with this problem.

The basic idea is illustrated by the twistor formulation of the 3D massive point particle [165]. The mass-shell constraint is solved in terms of a pair of constrained 3D spinor variables, and the phase-space is parametrized by these and their conjugate spinors which together constitute a pair of 3D twistors, i.e. spinors of the 3D conformal group $Sp(4;\mathbb{R})$. The constraints involve the mass and break the conformal invariance to Lorentz invariance. In the case of the 3D string, the Hamiltonian constraints may be solved in terms of a pair of commuting Majorana 3D spinor worldsheet fields. The phase space is then parametrized by these fields and their canonical conjugates, which together constitute a pair of 3D twistor worldsheet fields. This new formulation of the 3D string should have some similarities to the Hamiltonian form of the Green-Schwarz (GS) superstring action. As in that case, there are Lorentz invariant spinorial constraints (albeit with commuting spinors instead of anticommuting spinors) which are of mixed first and second class. Moreover, the second class constraints cannot be separated from the first class constraints in a Lorentz covariant way, and this greatly complicates any attempt at Lorentz covariant quantization, as it does for the GS superstring.

Intellectual Merit

The goal of this project is to further the understanding of several outstanding contemporary problems in theoretical physics, hence to elucidate the structure of the real world. Many of these problems have originated in intensive, worldwide research, both experimental and theoretical, related to supersymmetric quantum mechanical systems, in general, and to string theory, in particular.

Superstring theory is a model proposed to unify all the forces in Nature, and may very well have real-world applications involving exotic forms of matter. The model possesses remarkable mathematical structure, and suggests exciting new ways to view and understand the physical Universe.

A large class of important problems concerning the symmetries, geometry, and topology of superstring theories will be studied during this project. The principle investigators are masters of the techniques required for this research. The proposed activity suggests and explores very creative and original concepts, involving: mathematical deformations and non-commutative geometries in quantum mechanics; scale and conformal invariance and the behavior of real systems under changes in scale (renormalization group); galileon models in various spacetime dimensions; and supersymmetric models of semions, anyons, and strings in three-dimensional spacetime. The principle investigators consider these to be important topics with possible novel applications to several real physical systems.

For more than forty years, the principle investigators have made valuable contributions in their study of theoretical physics. This work has involved several international collaborations with many distinguished scientists. The research proposed here will continue these well-established scientific alliances with the goal to discover and pursue a deeper theoretical understanding of Nature.

The proposed activity is very important to advancing knowledge and understanding within elementary particle physics, as well as across different fields. The use of quantum theory to carry out quantitative calculations, and obtain predictions, is ubiquitous in all the physical sciences. The principal investigators are two of a relatively small number of experts uniquely qualified to carry out this research as evident from their training, their list of publications, and their research presentations.

Broader Impact

Teaching young scholars, thereby expanding the country's competitive STEM workforce, is an integral part of the career responsibilities of the principle investigators, and is fully symbiotic with their research programs and techniques. Thus the proposed activity will advance discovery and understanding while simultaneously promoting learning and improved STEM education and educator development at all levels, at a major research university, the University of Miami. The work will contribute greatly to enhance the human resource infrastructure for research and education at the University, which is in a region containing many underrepresented minorities in science, technology, engineering, and mathematics. These contributions will increase the economic competitiveness of the United States. The results will be disseminated broadly in colloquia and seminars, on internet web-pages, on internet archives, and in printed journals, to enhance scientific and technological understanding in its field. Society will benefit from the proposed activity in ways that are difficult to anticipate, but which clearly exist given the wide-ranging applications of the methodologies under development.

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Biographical Sketch: THOMAS L. CURTRIGHT

a. Professional Preparation

University of Missouri – Columbia	Physics	B.S. 1970
University of Missouri – Columbia	Physics	[redacted]. 1970
California Institute of Technology	Physics	[redacted]. 1977
University of California, Irvine	Physics	Postdoc 1976-1978
University of Chicago, Enrico Fermi Institute	Physics	Postdoc 1978-1980

b. Appointments

[1] Professor of Physics, University of Miami, Coral Gables, FL 33124, September 1988 — present.

[2] Scientific Associate, CERN, Geneva, Switzerland, May — November 2010.

[3] Alan Richards Fellow in Mathematics, University of Durham, UK, 31 May — 9 July 2007.

[4] Member, Institute for Advanced Study, Princeton, NJ 08540, January — July 2006.

[5] Visiting Scientist, Center for Theoretical Physics, MIT, Cambridge, MA 02139, April — May 1996.

[6] Member, Institute for Advanced Study, Princeton, NJ 08540, January — May 1990.

[7] Scientific Associate, CERN, Geneva, Switzerland, June — December 1987.

[8] Visiting Associate Professor of Physics, Yale University, New Haven, CT 06511, January — June 1987.

[9] Visiting Associate Professor of Physics, Institute for Theoretical Physics, State University of New York, Stony Brook, NY 11794, September 1986 — January 1987.

[10] Associate Professor of Physics (with tenure), University of Florida, Gainesville, September 1985 — August 1988.

[11] DOE Outstanding Junior Investigator and Assistant Professor of Physics, University of Florida, Gainesville, FL 32611, September 1980 — August 1985.

[12] Robert R. McCormick Fellow, The Enrico Fermi Institute, University of Chicago, Chicago, IL 60637, July 1978 — August 1980.

[13] Research Fellow, University of California, Irvine, CA 92717, September 1976 — June 1978.

c. Publications

Five Relevant to Recent Research:

[1] "Branched Hamiltonians and Supersymmetry" with C.K. Zachos, J.Phys. A47 (2014) 145201, arXiv:1311.6147 [math-ph]

[2] "Umbral Vade Mecum" with C.K. Zachos, Front. Physics 1 (2013) 15, arXiv:1304.0429 [math-ph]

[3] "A Galileon Primer" with [redacted]. Fairlie, arXiv:1212.6972 [hep-th]

[4] "Galileons and Naked Singularities" Phys.Lett. B716 (2012) 366-369, arXiv:1208.1205 [hep-th]

[5] "RG flows, cycles, and c-theorem folklore" with X. Jin and C.K. Zachos, Phys.Rev.Lett. 108 (2012) 131601, arXiv:1111.2649 [hep-th]

Five Others:

[1] *Quantum mechanics in phase space*, with [redacted]. Fairlie and C.K. Zachos, World Scientific (2005) DOI: 10.1142/9789812703507

[2] "Deforming Maps for Quantum Algebras" with C.K. Zachos, Phys. Lett. B243, 237-244 (1990) DOI: 10.1016/0370-2693(90)90845-W

[3] "Torsion and Geometrostasis in Nonlinear Sigma Models" with E. Braaten and C.K. Zachos, Nucl. Phys. B260, 630-688 (1985) DOI: 10.1016/0550-3213(85)90053-7

[4] "Conformally Invariant Quantization of the Liouville Theory" with C.B. Thorn, Phys. Rev. Lett. 48, 1309-1313 (1982) DOI: 10.1103/PhysRevLett.48.1309

[5] "Generalized Gauge Fields" Phys.Lett. B165, 304-308 (1985) DOI: 10.1016/0370-2693(85)91235-3

d. Synergistic Activities

[1] Physics conference organizer (1990, 1991, 1996, 2003, 2004-2014)
[2] Teaching and production of web-based learning materials
[3] Author of five books
[4] Speaker for colloquia and seminars (32 talks since 2005)
[5] Reviewer of books and articles submitted to journals

e. Collaborators and Other Affiliations

- Collaborators: D. Fairlie, Durham U., UK; X. Jin, L. Mezincescu, and A. Veitia, U. of Miami; C. Zachos, Argonne National Lab

- Advisors and Postdoctoral Sponsors: PhD thesis advisor, R. P. Feynman; postdoctoral sponsors, M. Bander, P. Freund, and Y. Nambu.

- Graduate Students: Andrzej Veitia, PhD 2010, currently postdoc at U. of Oregon; Xiang Jin, PhD 2012, currently employed by Nuance Corporation, Mahwah, NJ

Biographical Sketch: LUCA MEZINCESCU

a. Professional Preparation:

University of Bucharest, Romania.	Physics	[redacted]. 1969.
University of Bucharest, Romania.	Physics	[redacted]. 1978.
University of Texas, Austin	Theory Group	1983-1986

b. Appointments: Postgraduate

University of Miami	Professor	1996-
University of Bonn, Germany	Visiting Scientist Sabatical year	1995
University of Miami	Associate Professor	1990-96
CERN	Visitor	1991 Summer
University of Miami	Assistant Professor	1986-90
SUNY, Stony Brook	Visitor	1986 Winter
SLAC	Visitor	1984,87 Summer
Los Alamos Natl.Lab.	Visitor	1984,87 Summer
University of Texas at Austin, Theory Group	Post Doctoral Research Associate	1983-86
University of Michigan-Ann Arbor	Visiting Physicist	1983 Spring
University of California-Davis	Visiting Lecturer	1983 Winter
SLAC	Visiting Physicist	1982-83
Institute of Physics and Nucl. Eng., Bucharest, Romania	Research Fellow	1979-81
JINR-Dubna, USSR	Research Fellow	1978-79

Graduate

Institute of Physics and Nucl. Eng., Bucharest, Romania	Research Fellow	1977-78
[redacted]. Dubna USSR	Research Fellow	1974-75
[redacted]. Dubna USSR	Junior Research Fellow	1973-74
Institute of Physics	Res. Fellow	1972-77
Bucharest, Romania	Jun. Res. Fellow	1969-72

c. Publications

5 publications relevant to proposed research:

L. Mezincescu, A. J. Routh and P. K. Townsend, "Supertwistors and massive particles," Annals Phys. **346**, 66 (2014) [arXiv:1312.2768 [hep-th]].

L. Mezincescu, P. K. Townsend, "Anyons from Strings," Phys. Rev. Lett. **105**, 191601 (2010) , http://arxiv.org/pdf/1008.2334

L. Mezincescu, P. K. Townsend, "Semionic Supersymmetric Solitons," J. Phys. A **A43**, 465401 (2010), http://arxiv.org/pdf/1008.2775

A. Beylin, T. Curtright, E. Ivanov, L. Mezincescu, "Generalized N = 2 Super Landau Models," JHEP **1004**, 091 (2010), http://arxiv.org/pdf/1003.0218

T. Curtright, X. Jin, L. Mezincescu, D. Fairlie, C. K. Zachos, "Classical and Quantal Ternary Algebras," Phys. Lett. **B675**, 387-392 (2009),http://arxiv.org/pdf/0903.4889

5 other publications:

D. R. T. Jones and L. Mezincescu, "The Chiral Anomaly and a Class of Two-loop Finite Supersymmetric Theories", Phys. Lett. *138B*, 293-295 (1984).

M. Henneaux and L. Mezincescu, "A σ Model Interpretation of Green-Schwarz Covariant Superstrings", Phys. Lett. *152B*, 340-342 (1985).

M. T. Grisaru, P. Howe, L. Mezincescu, B. Nilsson and P. K. Townsend, "$N = 2$ Superstrings in a Supergravity Background", Phys.Lett. *B162*, 116-120 (1985).

L. Mezincescu and P. K. Townsend, "Stability at a Local Maximum in Higher Dimensional Anti-De Sitter Space and Applications to Supergravity", Annals Phys. *160*, 406-419, (1985).

L. Mezincescu and R. I. Nepomechie, "Integrability of Open Spin Chains with Quantum Algebra Symmetry", Int'l J. Mod. Phys. *A6*, 5231-5248 (1991), Addendum, *A7*, 5657-5659 (1992).

d. Synergistic Activities

Taught class of Physics for Music Engineers, 1994 - development of pedagogical methods.

Co-Organizer conference *Quantum Field Theory, Statistical Mechanics, Quantum Groups and Topology, Miami 1991.* - Member in the steering committee of *Miami (2004-2011)- A topical conference on elementary particle physics and cosmology*- service to the scientific community.

Member of the Advisory Board of various Conferences -transfer of knowledge

e. Collaborators and Other Affiliations

Collaborators

T.L. Curtright (Miami), E.I.Ivanov (Dubna, Russia), [redacted] (Cambridge U.K.), A Beylin (Miami),Xiang Jin (Miami), David Fairlie (Durham U., Dept. of Math. UK), Cosmas K. Zachos (Argonne, HEP).

Graduate and Postdoctoral Advisors

V. Novacu (deceased), V.I. Ogievetsky (deceased), S. Drell (SLAC), L. Susskind (Stanford), S. Weinberg (Austin).

Thesis Advisor and Postgraduate Sponsor

A. Beylin (2011) -Russia . A total 3 graduate, 5 postdoctoral scholars sponsored.

Biographical Sketch: COSMAS K ZACHOS

a. Professional Preparation

Princeton University	Magna Cum Laude	Physics	AB 1974
California Institute of Technology		Physics	[redacted]. 1979
University of Wisconsin		Physics	Postdoc 1979-1981
Fermilab		Physics	Postdoc 1981-1983

b. Appointments

[1] STA Physicist (Special theory advisor), High Energy Physics Division, Argonne National Laboratory 2014–present. Physicist, 1986–2014. Assistant Physicist, 1983–1986

[2] Research associate in theoretical physics, Fermilab, 1981–1983

[3] Research associate in theoretical physics, University of Wisconsin–Madison, 1979–1981

[4] Visiting appointments at IAS (Princeton) 1983; CINVESTAV (Mexico) 1985; Durham Univ (UK) 1987; OSU 1990; Yukawa Institute (Kyoto) 1994

c. Publications

Five Relevant to Recent Research:

[1] "Branched Hamiltonians and Supersymmetry" with T Curtright, J.Phys. A47 (2014) 145201, arXiv:1311.6147 [math-ph]

[2] "Umbral Vade Mecum" with T Curtright, Front. Physics 1 (2013) 15, arXiv:1304.0429 [math-ph]

[3] "RG flows, cycles, and c-theorem folklore" with X Jin and T Curtright, Phys.Rev.Lett. 108 (2012) 131601, arXiv:1111.2649 [hep-th]

[4] "Ternary Plots for Neutrino Mixing Visualization" EPL 99 (2012) 11001, arXiv:1205.4772

[5] "Renormalization Group Functional Equations" with T Curtright, Phys. Rev. D83 (2011) 065019, arXiv:1010.5174 [hep-th]

Five Others:

[1] "A Concise Treatise on Quantum Mechanics in Phase Space" with D Fairlie and T Curtright, [book] (Imperial Press, 2014)

[2] "Deforming Maps for Quantum Algebras" with C.K. Zachos, Phys. Lett. B243, 237-244 (1990) DOI: 10.1016/0370-2693(90)90845-W

[3] "Torsion and Geometrostasis in Nonlinear Sigma Models" with E. Braaten and C.K. Zachos, Nucl. Phys. B260, 630-688 (1985) DOI: 10.1016/0550-3213(85)90053-7

[4] "Trigonometric Structure Constants for New Infinite Dimensional Algebras" with D Fairlie & P Fletcher, Phys. Lett. 218B (1989) 203-206

[5] "Infinite-Dimensional Algebras, Sine Brackets, and SU(∞)" with D Fairlie, Phys. Lett. 224B (1989) 101-107

d. Synergistic Activities

[1] Principal Organizer of Physics Workshops, 1984, 1986, 1990, 1996, 2003, and Co-editor of their Proceedings

[2] Member of the APS Heineman Prize selection committee, 2011–2012

[3] Author of four books

[4] Speaker for colloquia (25 colloquia) and numerous seminars and conference presentations

[5] Associate Editor of Frontiers in Mathematical Physics, 2013—present. Member of the Editorial Board of Journal of Physics A, 1994–1998; presently, member of the Advisory Panel to the same journal. Member of the Editorial Board of ISRN Algebra. Refereed/adjudicated 46 papers since November 2009, and edited comparable number of papers. Referee of research proposals for numerous US Universities, NSF, DOE, NSERC, PSC-CUNY, and GNSF.

e. Collaborators and Other Affiliations

- Collaborators:T. Curtright, UK; X. Jin, and L. Mezincescu, U. of Miami; D. Fairlie, Durham U.

- Advisors: PhD thesis advisor, J. H. Schwarz; Teaching Assistant at Caltech for Particle Physics (Ph231 – R. P. Feynman) and for public course by M. Gell–Mann

- Given lecture series at UW-Madison, 1980; and Fermilab, 1983; CINVESTAV, Mexico City, 1985; OSU, 1989, 1990; Advanced summer course on phase-space quantization, Ph580, Koç University, Bosporus, Turkey, 2010; Boğaziçi University, 2014.